MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

ETN3096–DIGITAL SIGNAL PROCESSING

(LE / EE / CE / NE / TE)

12 JULY 2023
9.00 A.M – 11.00 A.M.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 10 pages including cover page and appendices, with 4 questions only.

2. Attempt **ALL FOUR** questions. The distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

Question 1

(a) Determine, with justification, if the discrete-time system $y[n] = x[n]e^{-n}$ is
(i) linear, [5 marks]
(ii) time-invariant. [5 marks]

(b) Consider the system shown in Figure Q1.

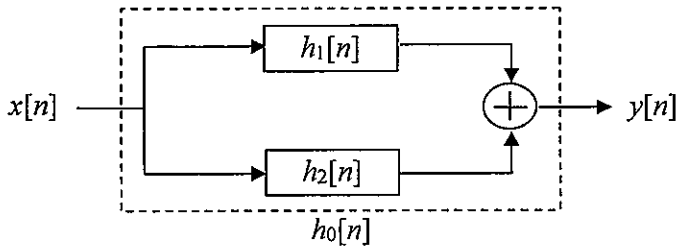

Figure Q1

(i) Given that $h_1[n] = 3\delta[n] + 5\delta[n-2]$, $h_2[n] = u[n] - u[n-3]$, and the impulse response of the overall system $h_0[n] = [\ldots\ 0\ \ \underline{a}\ \ b\ \ c\ \ 0\ \ldots]$, find the values a, b and c. [5 marks]
(ii) Using your answer in (i), find $y[n] = h_0[n] * x[n]$ using the table method if $x[n] = \delta[n] + 2\delta[n-1]$. [6 marks]

(c) A continuous-time signal is given by $x(t) = \cos 300t + \sin 800t$. What is the minimum sampling frequency required for converting $x(t)$ to its discrete-time representation without aliasing? [4 marks]

Question 2

(a) A causal system has a linear constant coefficient difference equation (LCCDE) given by

$$y[n] + 3y[n-1] + 2y[n-2] = 2x[n] - x[n-1]$$

(i) Determine the transfer function $H(z)$ for the LCCDE. [3 marks]
(ii) Find the region of convergence (ROC) of the z-transform for $H(z)$. [4 marks]
(iii) Is the system realizable? Justify your answer. [3 marks]

(b) Use direct discrete Fourier transform (DFT) to compute the 4-point DFT $X[k]$ of $x[n] = [\underline{3}\ \ 1\ \ 1\ \ 0]$. [9 marks]

(c) Assume that a complex addition takes 1μs and a complex multiplication takes 2μs. Compute the time required for direct evaluation of a 512-point DFT as well as the time required for evaluation of a 512-point DFT through fast Fourier transform (FFT). [6 marks]

Continued...

Question 3

(a) Design a 5-tap finite impulse response lowpass filter with a cutoff frequency of 2000 Hz and a sampling rate of 10,000 Hz.

(i) Calculate the filter coefficients when a rectangular window function is used. [10 marks]

(ii) Determine the transfer function and difference equation of the filter. [4 marks]

(iii) State why the designed filter is affected by the Gibbs effect. Suggest a remedy to the problem. [4 marks]

(b) (i) Consider the following mean square error for the Wiener filter

$$J = 60 - 40w + 30w^2$$

What is the optimal value of w? What is the corresponding value of J? [6 marks]

(ii) In a least mean square algorithm,

$$y(n) = w(n)x(n)$$
$$e(n) = d(n) - y(n)$$
$$w(n+1) = w(n) + 0.04e(n)x(n)$$

where $x(n)$ is the noise, $d(n)$ is the signal contaminated with noise and $w(n)$ is the filter coefficient. n is the sample index (discrete-time index). $y(n)$ and $e(n)$ are signals as defined by the equations above. Given that $w(0) = 0.3$, $x(0) = 0.5$ and $d(0) = 1$, calculate $w(1)$. [6 marks]

(iii) What is the step size used in part (ii)? State what will likely happen if the step size is decreased significantly. [3 marks]

Question 4

An infinite impulse response (IIR) filter has a transfer function given by

$$H(z) = \frac{0.6 + 0.345z^{-1} + 0.1z^{-2} + 0.4z^{-3}}{(1 - 0.1z^{-1})(1 + 0.8z^{-1} + 0.3z^{-2})}$$

(i) Draw the Direct Form I realization of the filter. [5 marks]

(ii) Draw the Direct Form II realization of the filter. [5 marks]

(iii) Compare the complexity of the Direct Form I and Direct Form II implementations in terms of the number of delays. [3 marks]

(iv) If the filter coefficients are to be quantized to 2 decimal places, evaluate the effect on the Direct Form I and Direct Form II implementations. [4 marks]

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APPENDIX

Discrete-time Fourier transform

Properties

Property	$x[n], y[n]$	$X(e^{j\Omega}),\ Y(e^{j\Omega})$
Linearity	$ax[n]+by[n]$	$aX(e^{j\Omega})+bY(e^{j\Omega})$
Time shifting	$x[n-n_0]$	$e^{-j\Omega n_0}X(e^{j\Omega})$
Frequency shifting	$e^{j\Omega_0 n}x[n]$	$X(e^{j(\Omega-\Omega_0)})$
Time reversal	$x[-n]$	$X(e^{-j\Omega})$
Differentiation	$n^k x[n]$	$(j)^k \frac{d^k}{d\Omega^k} X(e^{j\Omega})$
Convolution	$x[n]*y[n]$	$X(e^{j\Omega})Y(e^{j\Omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}(X(e^{j\theta}) * Y(e^{j\Omega}))$

Common pairs

$x[n]$	$X(e^{j\Omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\Omega n_0}$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega+2\pi k)$
$e^{j\Omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\Omega-\Omega_0+2\pi k)$
$u[n]$	$\frac{1}{1-e^{-j\Omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\Omega+2\pi k)$
$a^n u[n],\ \lvert a\rvert<1$	$\frac{1}{1-ae^{-j\Omega}}$
$(n+1)a^n u[n],\ \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\Omega})^2}$
$\frac{\sin\Omega_c n}{\pi n}$	$X(e^{j\Omega}) = \begin{cases}1, & \lvert\Omega\rvert<\Omega_c\\ 0, & \Omega_c<\lvert\Omega\rvert\le\pi\end{cases}$
$x[n]=\begin{cases}1, & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$	$\frac{\sin\Omega(M+1)/2}{\sin\Omega/2}e^{-j\Omega M/2}$

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z-transform

Properties

Properties	Sequence	z-transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R_x R_{x1} R_{x2}
Linearity	$ax_1[n]+bx_2[n]$	$aX_1(z)+bX_2(z)$	Contains $R_{x1} \cap R_{x2}$
Time shifting	$x[n-m]$	$z^{-m} X(z)$	R_x except for the possible addition or deletion of the origin or infinity.
Multiplication by an exponential sequence	$a^n x[n]$	$X(z/a)$	$\lvert a\rvert R_x$
Differentiation	$nx[n]$	$-z\frac{dX(z)}{dz}$	R_x except for the possible addition or deletion of the origin or infinity.
Conjugate	$x^*[n]$	$X^*(z^*)$	R_x
Time reversal	$x[-n]$	$X(z^{-1})$	$1/R_x$
Convolution	$x_1[n]^* x_2[n]$	$X_1(z)\,X_2(z)$	Contains $R_{x1} \cap R_{x2}$

Common pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

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Finite Impulse Response (FIR) Filters

Ideal impulse responses for standard FIR filters

Filter Type	Ideal Impulse Response $h(n)$

Lowpass:
$$h(n)=\begin{cases}\dfrac{\Omega_c}{\pi}, & n=0\\ \dfrac{\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$$

Highpass:
$$h(n)=\begin{cases}\dfrac{\pi-\Omega_c}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_c n)}{n\pi}, & n\neq 0\end{cases}$$

Bandpass:
$$h(n)=\begin{cases}\dfrac{\Omega_H-\Omega_L}{\pi}, & n=0\\ \dfrac{\sin(\Omega_H n)-\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$$

Bandstop:
$$h(n)=\begin{cases}\dfrac{\pi-(\Omega_H-\Omega_L)}{\pi}, & n=0\\ \dfrac{-\sin(\Omega_H n)+\sin(\Omega_L n)}{n\pi}, & n\neq 0\end{cases}$$

FIR filter length estimation using window functions

Window Type	**Window Function** $w(n), -M \le n \le M$
Rectangular	1
Hanning	$0.5+0.5\cos\left(\dfrac{n\pi}{M}\right)$
Hamming	$0.54+0.46\cos\left(\dfrac{n\pi}{M}\right)$
Blackman	$0.42+0.5\cos\left(\dfrac{n\pi}{M}\right)+0.08\cos\left(\dfrac{2n\pi}{M}\right)$

Normalized Transition Width $\Delta f = \dfrac{|f_{stop}-f_{pass}|}{f_{sampling}}$

Type of Window	**Window Length N**	**Stopband Attenuation (dB)**	**Passband Ripple (dB)**
Rectangular	$N = 0.9/\Delta f$	21	0.7416
Hanning	$N = 3.1/\Delta f$	44	0.0546
Hamming	$N = 3.3/\Delta f$	53	0.0194
Blackman	$N = 5.5/\Delta f$	74	0.0017

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Bilinear Transformation (BLT)

1. Frequency prewarping
Let ω_a denote the analog frequency marked on the $j\omega$ -axis on the s-plane, and ω_d denote the digital frequency marked on the unit circle in the z-plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right),\ \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

and $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

2. Prototype transformation using the lowpass prototype $H_P(s)$
From lowpass to lowpass: $H(s) = H_P(s)\big|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass: $H(s) = H_P(s)\big|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass: $H(s) = H_P(s)\big|_{s=\frac{s^2+\omega_0^2}{sW}}$

From lowpass to bandstop: $H(s) = H_P(s)\big|_{s=\frac{sW}{s^2+\omega_0^2}}$

where ω_a denotes the analog frequency, $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s)\big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

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Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_s = \dfrac{\omega_{as}}{\omega_{ap}}$
Highpass: ω_{ap}, ω_{as}	$\nu_s = \dfrac{\omega_{ap}}{\omega_{as}}$
Bandpass: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_s = \dfrac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$
Bandstop: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_s = \dfrac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge

ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband

ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband

ω_0, geometric center frequency

Closed-Form Expression for Some Useful Series

$$\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$$

$$\sum_{n=0}^{N-1} na^n = \frac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n = \frac{1}{2}N(N-1)$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \qquad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}$$

$$\sum_{n=0}^{N-1} n^2 = \frac{1}{6}N(N-1)(2N-1)$$

$$\sum_{n=N_1}^{N_2} a^n = \frac{a^{N_1-1} - a^{N_2}}{1-a}$$

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Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple A_p dB at the normalized passband frequency edge $v_p = 1$, and the stopband attenuation A_s dB at the normalized stopband frequency edge v_s, ε is the absolute ripple specification

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

Butterworth lowpass prototype order

$$n \geq \frac{\log_{10}\left(\frac{10^{0.1A_s} - 1}{\varepsilon^2}\right)}{2\log_{10}(v_s)}$$

Chebyshev lowpass prototype order

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\frac{10^{0.1A_s} - 1}{\varepsilon^2}}\right)}{\cosh^{-1}(v_s)}$$

where $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$

3-dB Butterworth Lowpass Prototype Transfer Functions ($\varepsilon = 1$)

n	$H_p(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2 + 1.4142s + 1}$
3	$\frac{1}{s^3 + 2s^2 + 2s + 1}$

Chebyshev Lowpass Prototype Transfer Functions with 1dB Ripple ($\varepsilon = 0.5088$)

n	$H_p(s)$
1	$\frac{1.9652}{s + 1.9652}$
2	$\frac{0.9826}{s^2 + 1.0977s + 1.1025}$
3	$\frac{0.4913}{s^3 + 0.9883s^2 + 1.2384s + 0.4913}$

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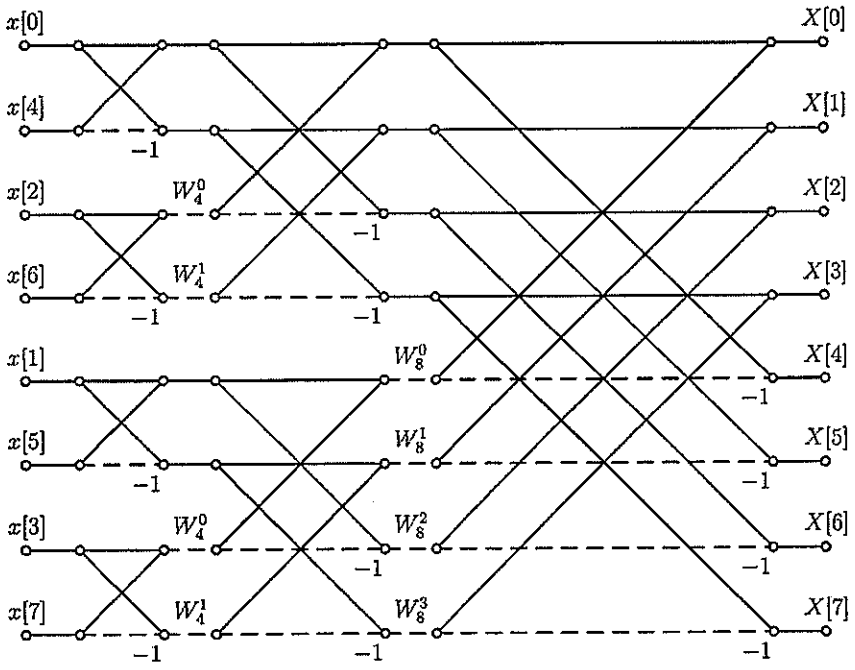
Discrete Fourier Transform

Properties

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time fast Fourier transform



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